Assessment Standard: 12.4.1(c)
Circle Geometry (Part 4)

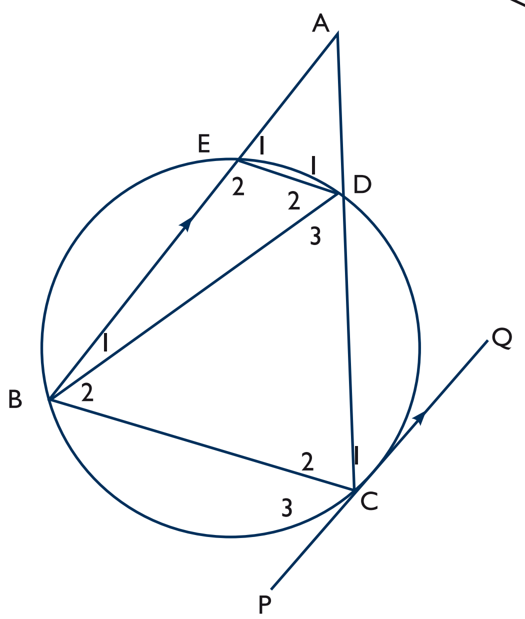
This one is slightly harder:

Think:
Is rad ⊥ tang?
Is tan chord theorem?

Rider 2

PQ is a tangent to the circle at C
AEB and ADC are straight lines
PQ || AB

Think:
i) Z alternate
ii) F corresponding
iii) '⊔' cointerior



Prove:

a) $\hat{A} = \hat{B}_2$ b) $\hat{B}_1 + \hat{B}_2 = \hat{D}_3$ c) $\hat{D}_3 = \hat{D}_1$

a) $\hat{B}_2 = \hat{C}_1$ (tan chord CD) (APPLY TAN CHORD)

$\hat{C}_1 = \hat{A}$ (alternate ∠s AB|| CQ; (Z shape))

so $\hat{B}_2 = \hat{A}$

can you see how we used $\hat{C}_1$ as the link or stepping stone?

b) $\hat{B}_1 + \hat{B}_2 = \hat{C}_3$ (alternate ∠s AB|| CP

$\hat{C}_3 = \hat{D}_3$ (tan chord CB)

so $\hat{B}_1 + \hat{B}_2 = \hat{D}_3$

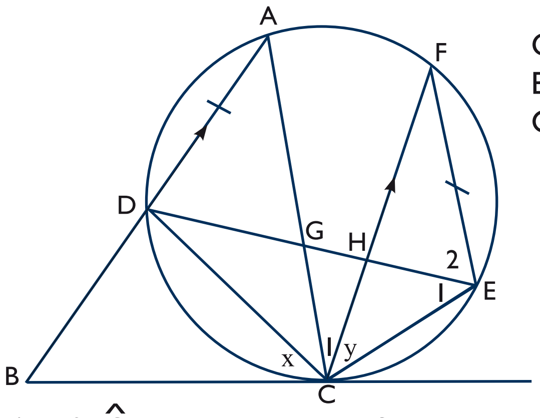
Again we used $\hat{C}_3$ as the link or stepping stone

c) $\hat{B}_1 + \hat{B}_2 = \hat{D}_1$ (ext ∠ cyclic quad EBCD)

$\hat{B}_1 + \hat{B}_2 = \hat{D}_3$ (proved above)

so $\hat{D}_1 = \hat{D}_3$

Rider 3



Given:
BC is a tangent to the circle at C
CF || AB and EF = AD

a) If $B\hat{C}D = x$, write down 3 other angles each equal to x (with reasons)

b) If $E\hat{C}F = y$, give a reason why $A\hat{C}D = y$

c) Prove $B\hat{D}C = G\hat{M}C$

a) $B\hat{C}D = x$ Given

$\hat{A} = x$ (tan chord theorem using DC)

$\hat{E}_1 = x$ (tan chord theorem using DC or ∠s same seg $\hat{A} = \hat{E}_1$)

$\hat{C}_1 = x$ (alternate ∠s BA || CF)

b) EF = AD (given)

EF subtends $E\hat{C}F = y$ and so $A\hat{C}D = y$ (equal chords subtend equal angles)

c) $B\hat{D}C = \hat{A} + A\hat{C}D$ (ext ∠ of a Δ)

$= x + y$

$G\hat{H}C = \hat{E}_1 + H\hat{C}E$ (ext∠ of a Δ)

$= x + y$

∴ $B\hat{D}C = G\hat{H}C$

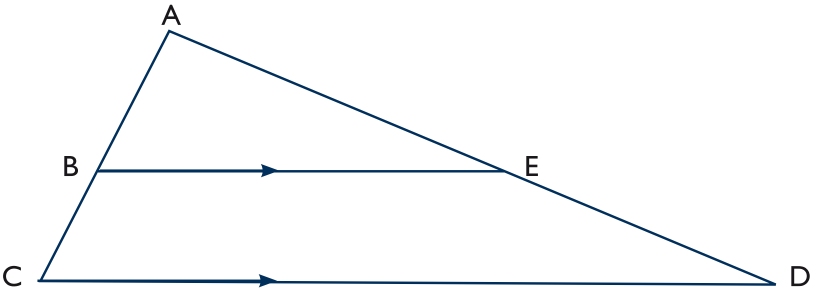
Can you see how much of this problem **needed <u>Grade 9</u> theory**?
So, please revise your basic parallel line and triangle theory as well.

Combined Grade 11 and Grade 12 Geometry

Perhaps one of the riders in your year-end examination will combine the proportion and similarity theory from Grade 11 with the Theory of Circles done in Grade 12.

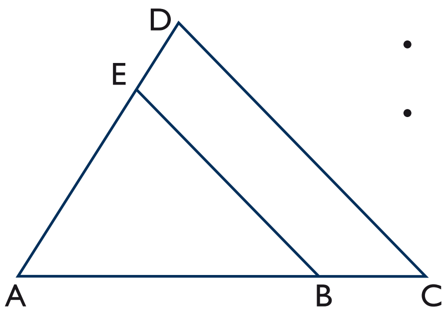
So, ... just a quick recap of Grade 11 Theory.

Theorem 1



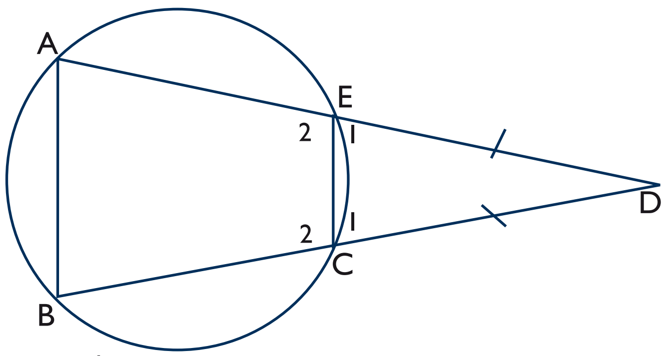
If in Δ ACD BE || CD then

- $\frac{AB}{BC} = \frac{AE}{ED} \Longleftrightarrow AB.ED = AE.BC$
- $\frac{AB}{AC} = \frac{AE}{AD} \Longleftrightarrow AB.AD = AE.AC$
- $\frac{BC}{CA} = \frac{ED}{DA} \Longleftrightarrow BC.DA = ED.CA$



and conversely if $\frac{AB}{BC} = \frac{AE}{ED}$ then BE || CD

So let's see how we can mix this into Grade 12



Given: Cyclic quad with A,B,C and E on circumference DE = DC

a) Prove EC || AB

b) If $ED = \frac{1}{3}AD$ and BC = 42 find CD

a) Let $\hat{E}_1 = x$

$\hat{E}_1 = \hat{C}_1 = x$ (Isos Δ DE = DC)

$\hat{E}_1 = \hat{B} = x$ (ext $\angle$ cyclic quad)

now $\hat{C}_1 = \hat{B} = x$

$\therefore$ EC || AB (equal corresp $\angle$ s)

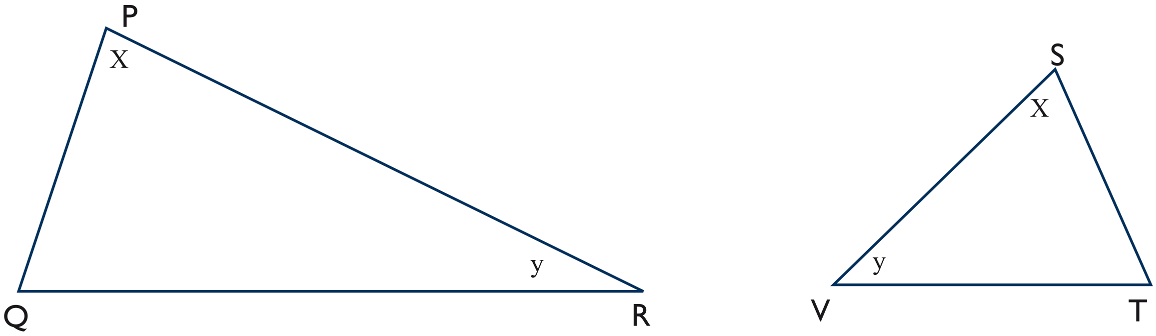
b) now $\frac{ED}{AD} = \frac{1}{3} = \frac{DC}{DB}$ (line || one side of Δ)

so $\frac{BC}{CD} = \frac{2}{1} = \frac{42}{CD}$

$\therefore$ CD = 21

Theorem 2

If two triangles are equiangular, their corresponding sides are in proportion.



Given Δ PQR and Δ SVT with $\hat{P} = \hat{S}$ and $\hat{R} = \hat{V}$ then $\frac{PQ}{ST} = \frac{PR}{SV} = \frac{QR}{TV}$

Remember

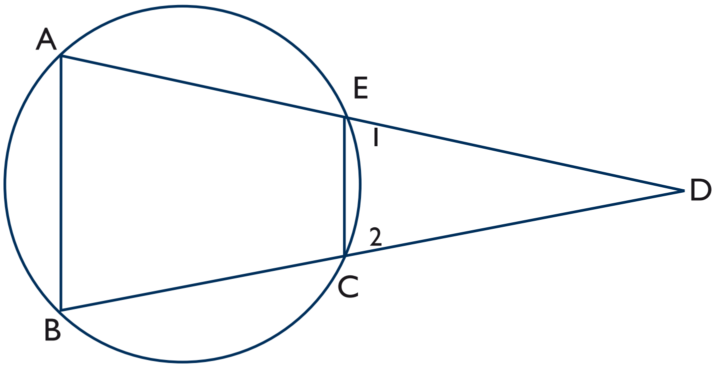
Δ PQR ||| Δ STV (similar Δs)

so

so $\frac{PQ}{ST} = \frac{QR}{TV} = \frac{PR}{SV}$ $\left(\frac{\text{first two}}{\text{first two}}, \frac{\text{last two}}{\text{last two}}, \frac{\text{outer two}}{\text{outer two}}\right)$

GETTING THE SIMILAR Δs IN THE RIGHT ORDER IS VITAL

Now look at this RIDER (freakishly like the previous one!)



Given : Cyclic quad ABCE

Prove DE.DA = DC.DB

In Δ DEC and Δ DAB

1) $\hat{D}$ is common

2) $\hat{E}_1 = \hat{B}$ ext $\angle$ cyclic quad

3) $\hat{C}_2 = \hat{A}$ ext $\angle$ cyclic quad

$\therefore$ Δ DEC ||| Δ DBA (AAA)

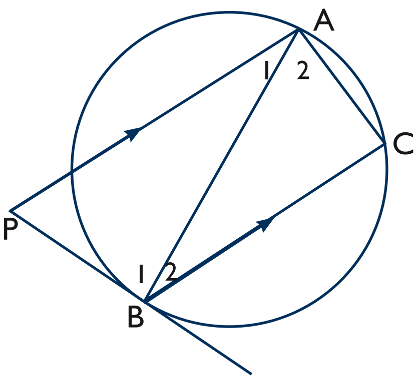
$\therefore \frac{DE}{DB} = \frac{DC}{DA} = \left(\frac{EC}{BA}\right)$ (similar Δs)

$\therefore$ DE.DA = DC.DB

See - not so bad!

Examples

1\.



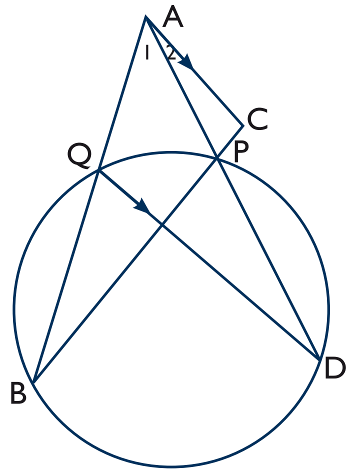
PB is a tangent and PA || BC

Prove that

a) Δ PAB ||| Δ ABC

b) AB^2 = PA.BC

2\.



AC || QD

a) Prove that Δ ABC ||| Δ PAB

b) Prove that AC^2 = BC.PC

c) Calculate AC if BC = 10,2 and PC = 3,4 units

1a) In Δ PAB and Δ ABC

$\hat{A}_1 = \hat{B}_2$ (alt $\angle$s PA || BC)

$\hat{B}_1 = \hat{C}$ (tan chord using BA)

$\hat{P} = \hat{A}_2$ (sum $\angle$s of a Δ)

$\therefore$ Δ PAB ||| Δ ABC (AAA)

b) $\underbrace{\frac{PA}{AB} = \frac{AB}{BC}} = \left(\frac{PB}{AC}\right)$ (similar Δs)

AB.AB = PA.BC

AB^2 = PA.BC

2a) $\hat{A}_2 = \hat{D}$ (alt $\angle$s AC || QD)

$\hat{D} = \hat{B}$ ($\angle$s same segment)

$\therefore$ $A_2 = \hat{B}$

In Δ ABC and Δ PAC

$\hat{C}$ is common

$\hat{B} = \hat{A}_2$ (proved)

so $\hat{A}_1 + \hat{A}_2 = \hat{P}$ sum $\angle$s of a Δ

so Δ ABC ||| Δ PAC (AAA)

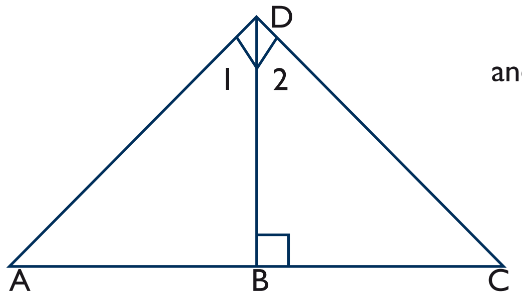
b) $\left(\frac{AB}{PA}\right) = \underbrace{\frac{BC}{AC} = \frac{AC}{PC}}$ (similar Δs)

so $AC^2 = BC.PC$

c) $AC^2 = 10,2 \times 3,4$

$AC = 5,9$ units

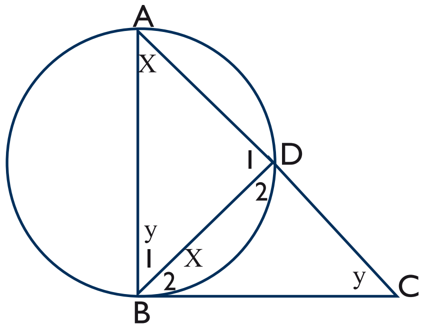
Now the last Grade 11 Theorems said that a perpendicular dropped from a right angle to a hypotenuse creates two triangles similar to each other and to the original triangle.



Δ ABD ||| Δ DBC ||| Δ ADC

and $AD^2 = AB \times AC$____(1)
$DC^2 = CB \times CA$___(2)
$DB^2 = AB \times BC$____(3)

Example



BC is a tangent
AB is a diameter
Prove $AB^2 = AD.AC$

Proof

$\hat{B}_1 + \hat{B}_2 = 90°$ (rad $\perp$ tang)

$\hat{D}_1 = 90°$ ($\angle$ in semi circle)

$\therefore$ Δ ABD ||| Δ ACB (perp from 90° to hypotenuse)

$\therefore$ $\underbrace{\frac{AB}{AC} = \frac{AD}{AB}} = \frac{BD}{CB}$ (similar Δs)

$\therefore$ $AB^2 = AC.AD$