Assessment Standard: 12.3.2
Circle Geometry (Part 2)

In Part 1 we covered the basics, the terminology and the first two theorems.

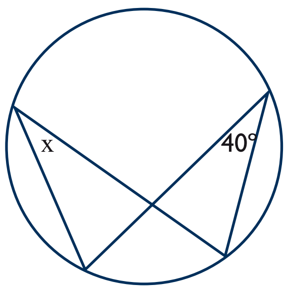
Now, we start with:

Theorem 3

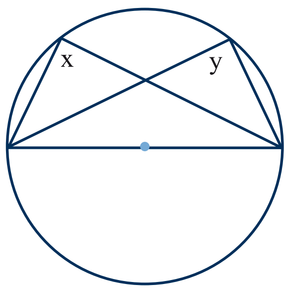
The angles subtended on the same arc in the same circle segment to the same circumference, are equal (∠s same segment)

Application:

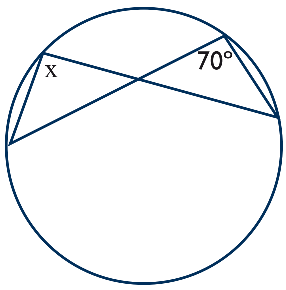
a) $x = 40°$ (∠s same segment)



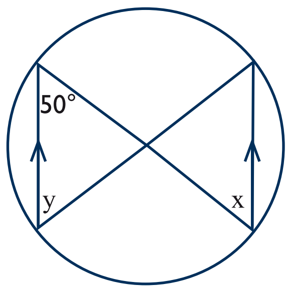
c) $x = y = 90°$ (∠s same segment)



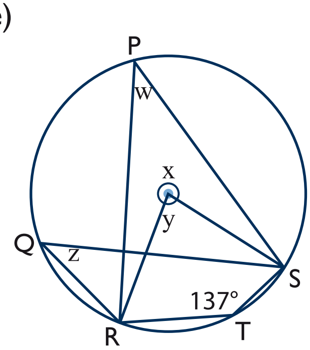
b) $x = 70°$ (∠s same segment)



d) $x = 50°$ (alt ∠s || lines)
$y = 50°$ (∠s same segment)



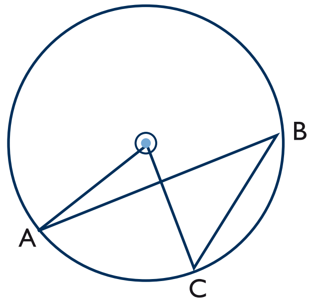
e)



$x = 274°$ (∠ at centre = 2∠ at circumference)
$y = 86°$ (∠s around a point)
$z = 43°$ (∠ at centre = 2∠ at circumference)
$w = 43°$ (∠s same segment)

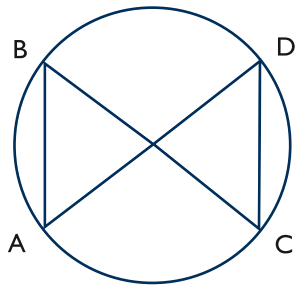
Some learners confuse these two situations

Case 1



Here $A\hat{O}C$ is double $A\hat{B}C$
(∠ at centre 2 ∠ at circumference)

Case 2

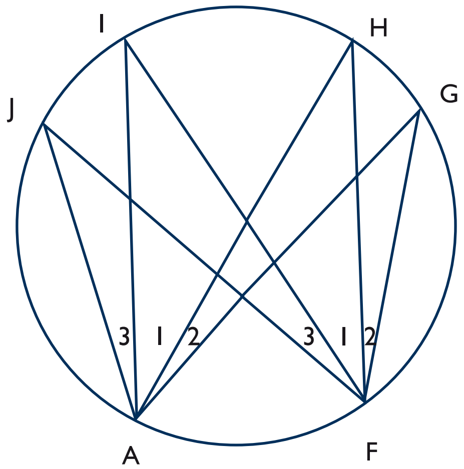


Here $A\hat{B}C = A\hat{D}C$
(∠s same segment)

Can you see that in Case 1 the arc AC subtends $A\hat{O}C$ to the centre and $A\hat{B}C$ to the circumference?

Can you see that in Case 2 the arc AC subtends $A\hat{B}C$ and $A\hat{D}C$ to the circle edge?

Look at these!



Can you see that:

1) $\hat{J} = \hat{I} = \hat{H} = \hat{G}$?
(both subtended / hung by arc AF)

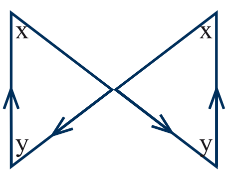
2) $\hat{A}_1 = \hat{F}_1$
(both subtended/ hung by arc HI)

3) $\hat{A}_2 = \hat{F}_2$?
(both subtended/ hung by arc HG)

4) $\hat{A}_3 = \hat{F}_3$?
(both subtended/ hung by arc JI)

If you can't see these turn the picture upside down. It helps.

Also try use your figures to trace out the 'Bow Tie':



the top two angles are equal and so are the bottom two

START / END

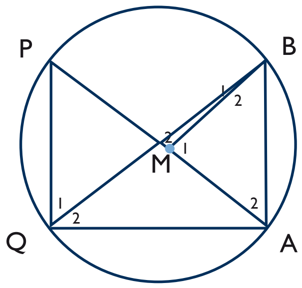
Let us see if we can mix up the application of these theorems so far.

A geometry problem is called a RIDER

To solve a RIDER apply the following steps:

1. Read the given information and respond to / use all key words
eg: If you have a circle centre mark off equal radii; if you have a midpoint of a chord mark in the perp. ; if you have equal radii mark in the equal angles of the isosceles triangle
2. Fill in all the information on the sketch - as well as all immediately apparent logical deductions
3. Look carefully at what you are asked to prove - if it is to prove two angles equal, start at the one and try to find a stepping-stone sequence to get to the other one via an indirect route

RIDER 1



Given circle centre M.

$\hat{B}_1 = 5°$ and $\hat{Q}_1 = 53°$ calculate with reasons

a) $\hat{Q}_2$

b) $\hat{M}_1$

c) $\hat{B}_2$

d) $\hat{P}$

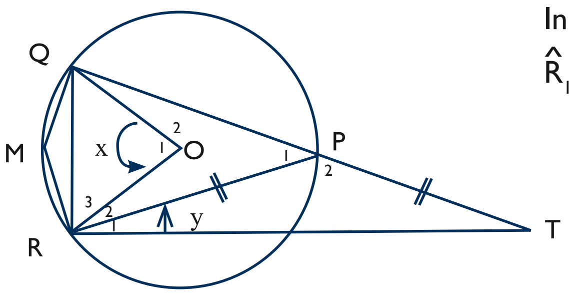
a) $\hat{Q}_2 = 37°$ ($\angle$ in a semi-circle; $\hat{Q}_1 + \hat{Q}_2 = 90°$)

b) $\hat{M}_1 = 74°$ ($\angle$ at centre 2 $\angle$ at circumference)

c) $\hat{B}_2 = 53° = \hat{A}_2$ ($\angle$s of Isos Δ MBA; MB = MA radii)

d) $\hat{P} = \hat{B}_1 + \hat{B}_2 = 58°$ ($\angle$s in same segment)

RIDER 2



In the figure O is given the centre of the circle with PT = PR

$\hat{R}_1 = y$ and $\hat{O}_1 = x$

1 Express x in terms of y

2 If TQ = TR and $x = 120°$ calculate the measure of

a) y

b) $\hat{R}_2$

3 Find $\hat{M}$

Solutions:

1. In Δ PRT

$\hat{T} = y$ (Isos Δ PR = PT given)

$\therefore$ $\hat{P}_1 = \hat{R}_1 + \hat{T} = 2y$ (ext $\angle$ of Δ)

but $\hat{O}_1 = 2\hat{P}_1$ ($\angle$ at centre = 2 $\angle$ at cirumference)

$\therefore$ $x = 2(2y)$

$\therefore$ $x = 4y$

2a) If $x = 120°$ and $x = 4y$, $y = 30°$

b) In Δ QTR, $\hat{T} = 30°$ and QT = RT (given)

$\therefore$ $T\hat{Q}R = T\hat{R}Q = 75°$ (Isos Δ)

now in Δ OQR OR = OQ (equal radii)

so $\hat{R}_3 = 30°$ ($\angle$s of Δ)

$\therefore$ $\hat{R}_2 = 15°$ (75° - 30° - 30°)

c) $\hat{O}_2 = 240°$ ($\angle$ around a point)

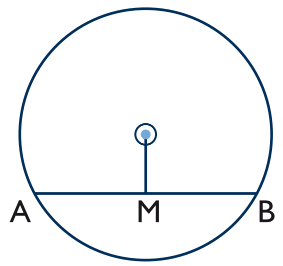
$\therefore$ $\hat{M} = 120°$ ($\angle$ at centre 2 $\angle$ at circ-reflex case)

Before we move on to the next theorems we must mention something here called a CONVERSE.

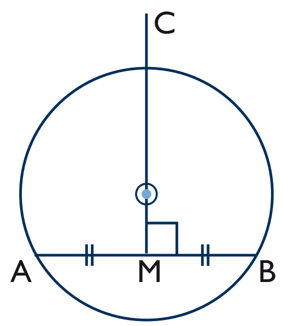
The CONVERSE of a theorem is what we get if we turn the theorem about or reverse it in order. We basically swop the known/given with the logical conclusion. So, in fact Theorem 1a) and b) are converses.

If OM $\perp$ AB then AM = MB

If AM = MB then OM $\perp$ AB



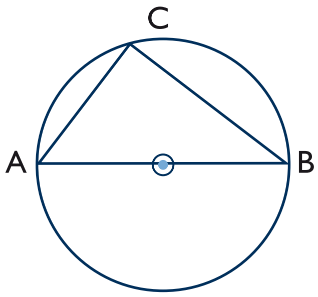
Theorem 1 has another converse which says that a perpendicular bisector of a chord must go through the centre O. So, if AM = MB and CM $\perp$ AB then centre O must lie on CM somewhere!

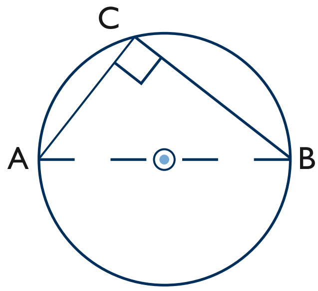


Another theorem that has a converse is the angle in the semi-circle one. Let's turn it around.

IF AB is diameter than $\hat{C} = 90°$

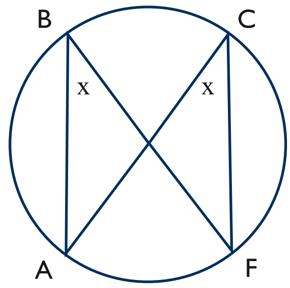
If $\hat{C} = 90°$ then AB is diameter





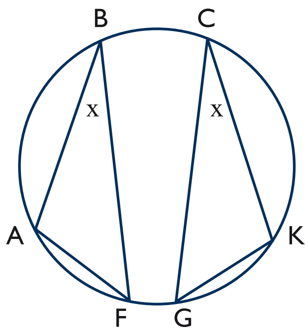
Sometimes a theorem doesn't have a converse, but it does have some further application.

We know $\hat{B} = \hat{C}$ because they are subtended by AF



Now here $\hat{B} = \hat{C}$ also.

Why? because they are subtended by equal chords AF and GK

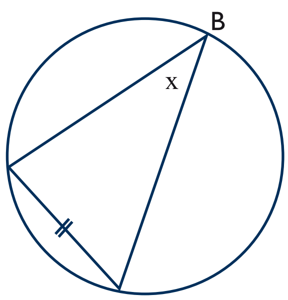


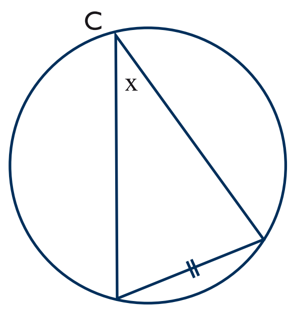
So if we see this in a RIDER we would say that $\hat{B} = \hat{C}$ (subtended by equal chords)

We can take this logic one step further.

What if we have two equal circles (same size/radius) and two chords the same length?

Well - can you guess that $\hat{B} = \hat{C}$ here too?





But, this only works in equal circles off equal chords!