Financial Maths

1. a) Simple Interest and Depreciation

$A = P(1 + in)$ or $A = P(1 - in)$

Compound Increase and Depreciation

$A = P(1 + i)^n$ or $A = P(1 - i)^n$

P is principal amount/amount at start/amount invested
n is number of periods for which amount is invested
i is the periodic interest rate
A is final amount

Remember . $P = \frac{A}{(1+i)^n} = A(1+i)^{-n}$ **formula for finding P**

. $n = \frac{\log \frac{A}{P}}{\log(1+i)}$ **formula for finding n**

Use formula for once off/random deposits and withdrawals.

b) Time Line

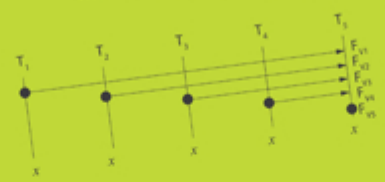
$i_E = 0,0825$ $i_E = 0,10$

At T_8 investment = $10\,000(1 + 0,0825)^4(1+0,10)^4 + 5\,500(1 + 0,0825)(1 + 0,10)^4 + 3\,000(1 + 0,10)^4$

R24 428,58

2. Future Value Annuity

Invest a certain amount at regular intervals - called Annuity
Make monthly payment at end of each month and this money gets interest



So if you save R400 everty month for 3 years at 12% p.a compounded monthly gives you.

$Fv = x\frac{[(1+i)^n - 1]}{i}$

= R17 230,75

Periodic interest must match time periods

3. Present Value Annuity - Repayment of Loan

Pay a certain amount at regular intervals to pay off a loan.
Amount to be repaid includes the interest on the loan amount.
Intial loan amount called the Present Value.

So Future Value of loan = Future Value of Annuity

$\text{Present Value} \times (1+i)^n = x\frac{[(1+i)^n - 1]}{i}$

$\text{Present Value} = x\frac{[(1+i)^n - 1]}{i(1+i)^n}$

$= \boxed{x\frac{[1-(1+i)^n]}{i}}$

Interest rate of 15% p.a compounded monthly for 5 years

So if loan of R150 000 taken at an

$Pv = 15\,000$

$150\,000 = x\frac{\left[1-\left(1+\frac{0,15}{12}\right)^{-60}\right]}{\frac{0,15}{12}}$

R 3 568,49 = x

Note: Paid 60 x 3568,49 = R214 109,37 to pay loan of R150 000.

4. Investments and Loan Options

Micro Lenders (loan sharks) - AVOID THEM!
If they charge you 10% compound interest per month this means in effect you pay:

$1 + i_E = (1+i)^n$
$i_E = (1+0,1)^{12} - 1$
$i_E = 2,138$

an effective annual interest rate of 214%!!

5. Which is better?

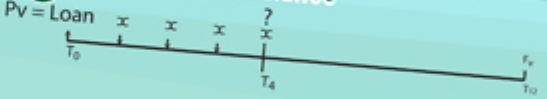
12,5% pa compounded monthly or 12,6% pa compounded quarterly

$\left(1+\frac{0,125}{12}\right)^{12} = 1 + i_E$ ✔
$0,1324 = i_E$
$r = 13,24\%$

$\left(1+\frac{0,126}{12}\right)^{4} = 1 + i_E$ ✘
$0,13207 = i_E$
$r = 13,21\%$

Must convert nominal rate to effective rate to be able to compare them.

6. Outstanding Balance



At T_4 Outstanding Balance = (loan + interest) - (payments + interest)

loan $(1+i)^4 - x\left[\frac{(1+i)^4-1}{i}\right]$

OR

We need to move all payments not made yet back to T_4 so

$OB_4 = x\frac{[1-(1+i)^{-8}]}{i}$