

TRANSFORMATION GEOMETRY

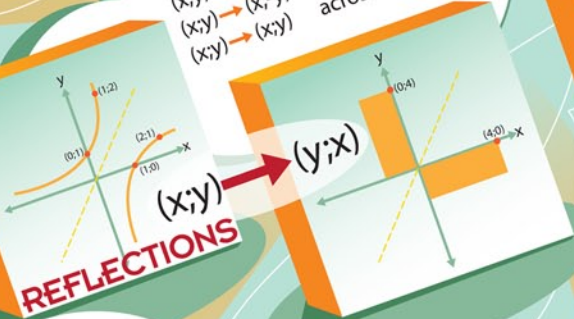
This topic includes transformations that preserve SHAPE and SIZE:

We can use transformations on a single point, a shape or a graph

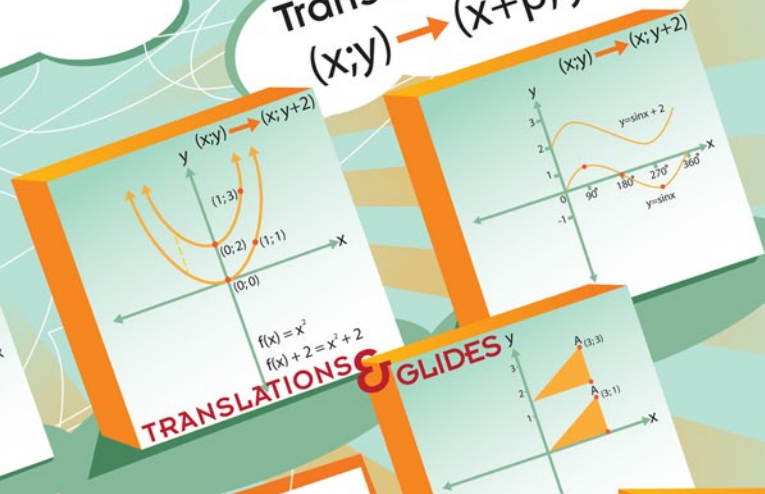
Translations (glides)
 $(x;y) \rightarrow (x+p; y+q)$

Reflections

$(x;y) \rightarrow (-x;y)$ across the y axis
 $(x;y) \rightarrow (x;-y)$ across the x axis
 $(x;y) \rightarrow (y;x)$ across the line $y=x$

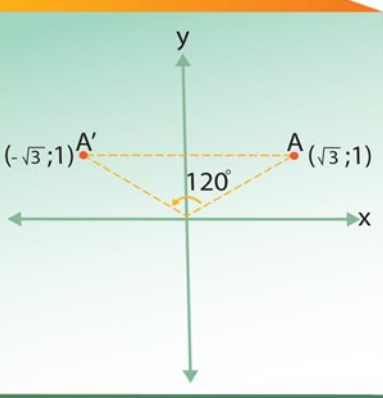


TRANSLATIONS & GLIDES



Rotations

$(x;y) \rightarrow (-y;x)$ about origin through 90° anti-clockwise
 $(x;y) \rightarrow (y;-x)$ about origin through 90° clockwise
 $(x;y) \rightarrow (-x;-y)$ about origin through 180°
 $(x;y) \rightarrow (x \cos \theta - y \sin \theta; y \cos \theta + x \sin \theta)$ about origin through θ



Given A(√3; 1)

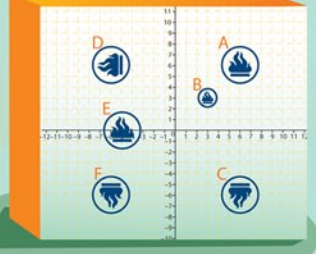
The transformed point A' is called the image of A
 We can describe the transformation in a number of ways:
 A' is a reflection of A in the y-axis $(x;y) \rightarrow (-x;y)$
 A' is a translation (glide) of A by moving $2\sqrt{3}$ units to the left
 $(x;y) \rightarrow (x - 2\sqrt{3}; y)$
 A' is a rotation of A about the origin through 120° anti-clockwise
 $(x;y) \rightarrow (x \cos 120^\circ - y \sin 120^\circ; y \cos 120^\circ + x \sin 120^\circ)$

ROTATIONS

Given B(1; 4)

B(-1; -4) is the image of B.
 B' is a translation of B down 8 units and across 2 units left $(x;y) \rightarrow (x-2; y-8)$
 B'' is a rotation of B about the origin through 180° $(x;y) \rightarrow (-x;-y)$
 B''' is a combined two-step rule
 B''' is a reflection of B across the y-axis followed by a reflection across the x-axis
 $(x;y) \rightarrow (-x; -y)$
 $(x;y) \rightarrow (-x; y)$
 $(x;y) \rightarrow (x; -y)$
 $(x;y) \rightarrow (x; y)$

Written as a single rule Transformation:



A to B is an **ENLARGEMENT** of scale factor $\frac{1}{2}$ $(x;y) \rightarrow (\frac{x}{2}; \frac{y}{2})$
 A to C is a **REFLECTION** in the x axis $(x;y) \rightarrow (x;-y)$
 A to D is a **ROTATION ANTI-CLOCKWISE** through an angle of 90° about the origin $(x;y) \rightarrow (-y;x)$
 A to E is a **TRANSLATION** down 6 and across 11 to the left $(x;y) \rightarrow (x-11; y-6)$
 A to F is a **ROTATION** about the origin through 180° $(x;y) \rightarrow (-x;-y)$
 C to D is a **REFLECTION** about the line $y=x$

Grade 12 theory concentrates on rotating about the origin through any angle

Use formula
 $(x;y) \rightarrow (x \cos \theta - y \sin \theta; y \cos \theta + x \sin \theta)$ anti-clockwise
 $(x;y) \rightarrow (x \cos (-\theta) - y \sin (-\theta); y \cos (-\theta) + x \sin (-\theta))$ clockwise

FIND A' :

$$x' = 2 \cos 60^\circ - 2 \sin 60^\circ = 1 - \sqrt{3}$$

$$y' = 2 \cos 60^\circ + 2 \sin 60^\circ = 1 + \sqrt{3}$$

$$\therefore A' (1 - \sqrt{3}; 1 + \sqrt{3})$$

IF A'' (-1-√3; 1-√3) FIND θ

see that $(-1 - \sqrt{3}; 1 - \sqrt{3}) = (-1 + \sqrt{3}; 1 - \sqrt{3})$
 so $(x;y) \rightarrow (-y;x)$
 so this is a rotation of A' through 90° anti-clockwise
 $\therefore \theta = 60^\circ + 90^\circ = 150^\circ$

Enlargements preserve **SHAPE** but not **SIZE**

1 IS AN ENLARGEMENT OF 2 BY SCALE FACTOR 2

The area of 1 is 4 times the area of 2
 The perimeter of 1 is 2 times the perimeter of 2

4 IS AN ENLARGEMENT OF 3 BY SCALE FACTOR $\frac{1}{2}$

The area of 4 is $\frac{1}{4}$ times the area of 3
 The perimeter of 4 is $\frac{1}{2}$ times the perimeter of 3

